SOLUTIONS OF IIT-JEE 2011
SET-3
PAPER - I

CHEMISTRY, PHYSICS AND MATHEMATICS

CHEMISTRY

1. (a) $^{27}_{13}A\ell \longrightarrow {}^{30}_{15}P + y$

$^{27}_{13}A\ell + ^{4}_{2}He \longrightarrow {}^{30}_{15}P + ^{1}_{0}n = y$

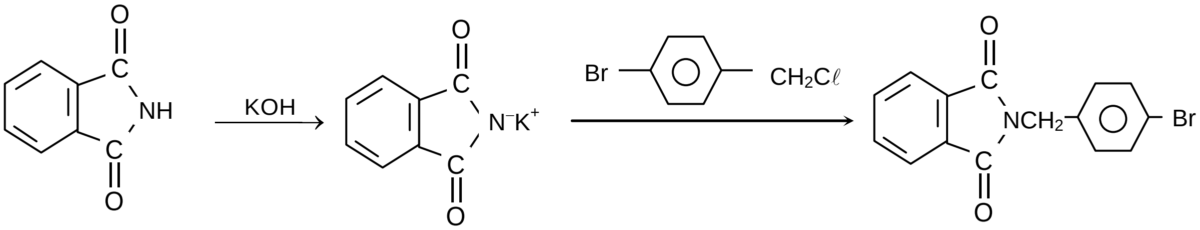
$^{27}_{13}A\ell + ^{4}_{2}He \longrightarrow {}^{30}_{14}Si + ^{1}_{1}H = x$

$^{30}_{15}P \longrightarrow {}^{30}_{14}Si + ^{0}_{1}n = z$

2. (c) $M = \dfrac{\text{Mass of solute} \times 1000}{\text{Molar mass solute} \times \text{volume of solution}(m\ell)}$

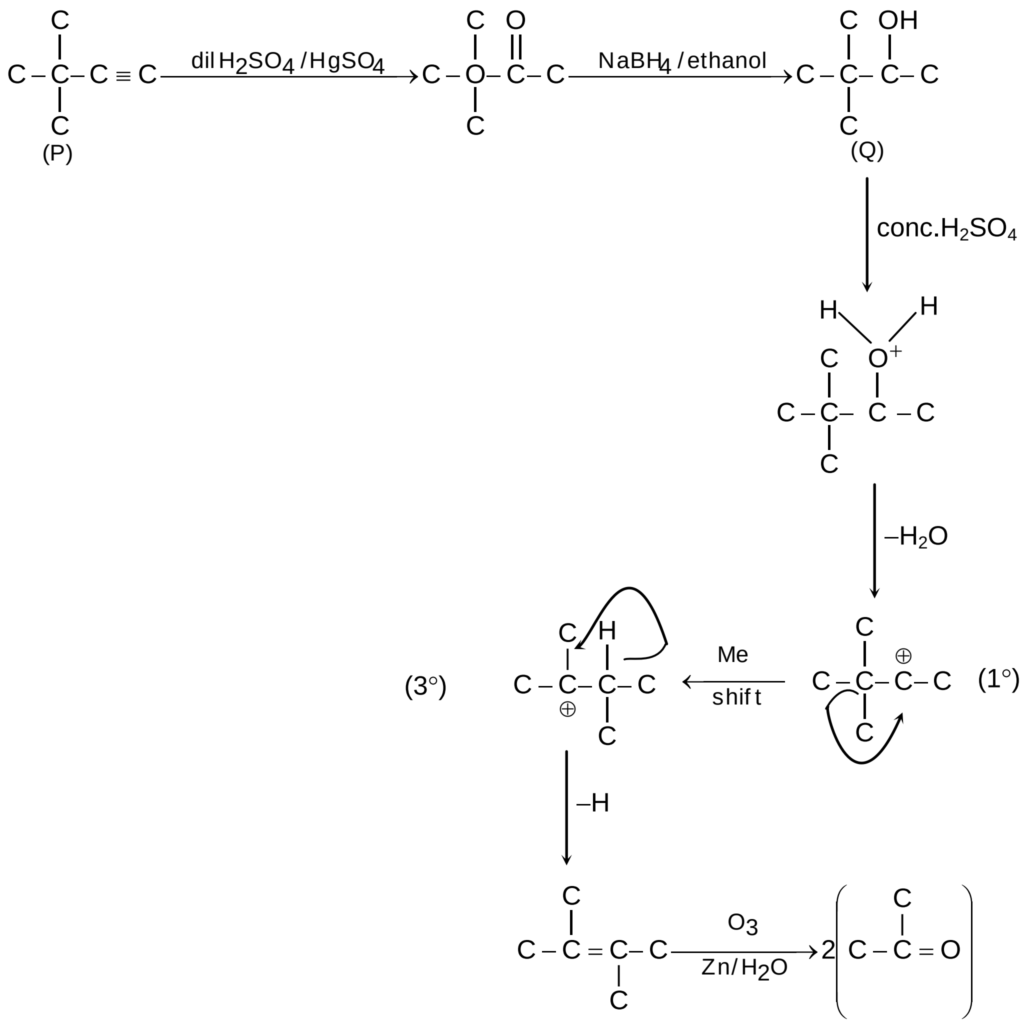
$= \dfrac{120}{60 \times \dfrac{1120}{1.15}} \times 1000 = 2.05\ M$

3. (d)
4. (c)
5. (a) Gabriel Pthalamide reaction



6. (d)
7. (b) $Ni^{+2} \longrightarrow 3d^8$
$C\ell \longrightarrow$ Weak ligand $\longrightarrow$ tetrahedral
$CN^{-1} \longrightarrow$ Strong Ligand $\longrightarrow$ square planar
8. (a,b,c,d)
9. (b,c)
10. (a,c,d) Cassarite $\longrightarrow SnO_2$ reduced by carbon.
11. (a,d)
12. (b) $\underset{(M)}{Cu} + \underset{(N)}{AgNO_3} \longrightarrow Cu(NO_3)_2 + Ag$
13. (a)
14. (c)

15. (d)



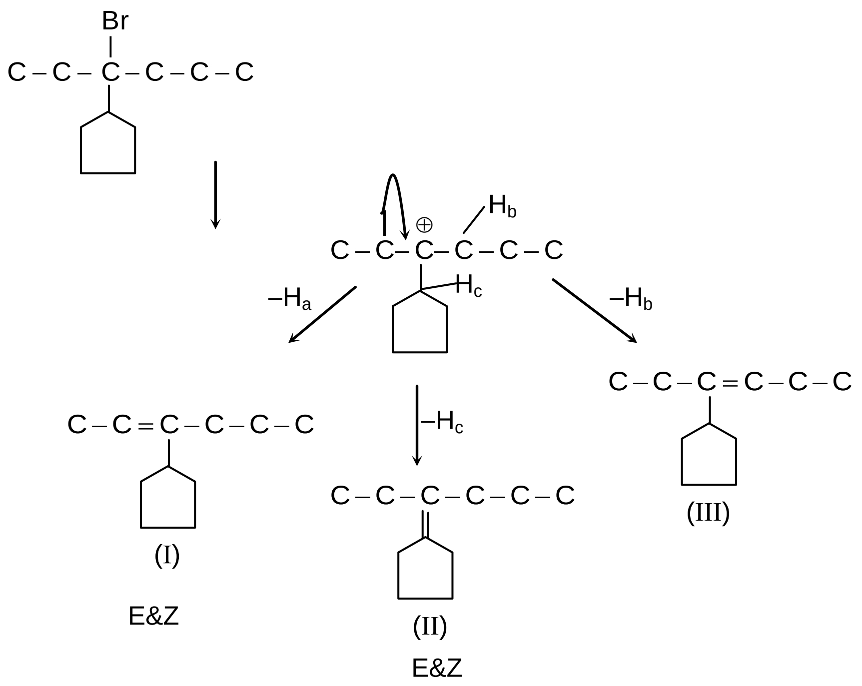
16. (b)

17. (9) n^2 = no. of orbitals = $3^2 = 9$

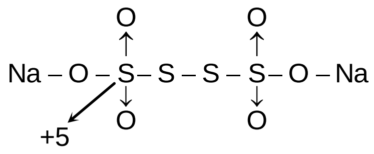
$M_s = -1/2$ (each orbital contain $1e^{\ominus}$)

$\therefore$ no. of mass $1e^{\ominus} = 9$

18. (5)



19. (5)



Change in oxidation state = 5 – 0 = 5

20. (6) $\frac{47}{100} \times \frac{(796 + 9 \times 18)}{75}$

21. (7) PV = nRT

$V = \frac{0.1 \times 0.082 \times 273}{0.32} = 7$

22. (5) $3Br_2 + 3Na_2CO_3 \longrightarrow 5NaBr + NaBrO_3 + 3CO_2$

23. (4) I.E = $h\nu - \phi$

for photo electric effect

$h\nu > \phi$

$h\nu = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{30010^{-9}} \times 6.24 \times 10^{18}$

= 4.1 eV

PHYSICS

24. (c) Area of projection = a^2

hence flux = Ea^2

25. (a) First Balmer line is given by

$$\frac{1}{\lambda_H} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \qquad \text{....(1)}$$

Similarly second spectral line of Balmer atom for He

$$\frac{1}{\lambda_{He}} = R(2^2)\left(\frac{1}{2^2} - \frac{1}{4^2}\right) \qquad \text{....(2)}$$

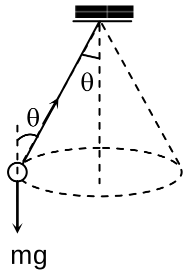
From (1) & (2)

$$\frac{1/6561}{1/\lambda_{He}} = \frac{5}{36}\frac{16}{(4)\,3} \Rightarrow \lambda_{He} = 1215\ \text{Å}$$

26. (d) $T \sin\theta = m\omega^2 L \sin\theta$

$324 = (.5)\,\omega^2(.5)$

$\Rightarrow \omega = 36$ rad/sec



27. (b) $\frac{x}{10} = \frac{52+1}{48+2}$

Values of end correction will be added

$\because$ when the string is tan t

Length will be increased

$\therefore x = 10\left(\frac{53}{50}\right)$

$x = 10.6\ \Omega$

$\therefore$ (b) is correct

28. (d) Initial P.E. stored

$U_I = \frac{1}{2}(2\mu F)V^2 \mu J$

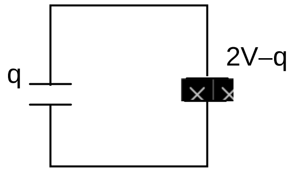
Initial charge

$Q_I = 2V$

Charges after the switch is changed position

$\frac{q}{2} = \frac{2V - q}{8}$

$\Rightarrow q = \frac{2}{5}V$



or New P.D = $\frac{q}{2} = \frac{V}{5}$

New P.E $U_f = \frac{1}{2}\left[2\left(\frac{V}{5}\right)^2 + 8\left(\frac{V}{5}\right)^2\right]$

$= \frac{1}{2}\left[\frac{2}{5}V^2\right]$ MJ

So loss in P.E = 80% of initial energy

29. (a) $f' = f\left[\frac{V - V_o}{V - V_s}\right]$

$= 8\left[\frac{320+10}{320-10}\right]$

$f' = 8.50$ kHz

30. (a) $W = \frac{nRT\Delta T}{\gamma - 1}$

Now $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\Rightarrow T_1(5.6)^{2/3} = T_2(.7)^{2/3} \Rightarrow T_2 = 4T_1$

$W = \frac{nR3T_1}{2/3} = \frac{9}{2}nRT_1 = \frac{9}{2}\frac{1}{4}RT_1 = \frac{9}{8}RT_1$

31. (a,c,d) $\frac{d\theta}{dt}$, i.e. Rate of flow of heat through each point will remain same

$\therefore$ (a) is true

(b) is false

Temperature difference across E and A will be minimum as their length is smallest

$4k.2A\left(\frac{4L}{4L}\right) = 3kA\left(\frac{4L}{4L}\right) + 5kA\left(\frac{4L}{4L}\right)$

$\therefore h_c = h_B + h_D$

$\therefore$(a,c,d) are correct

32. (a,d) When disc is not free to rotate its moment of inertia will count. Due to which angular velocity decreases i.e. in case A

$\therefore \omega_A < \omega_B$

But as angular position is same

Restoring torque will remain unchanged.

33. (a,b,c,d)

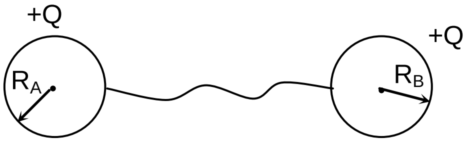
Inside conducting spheres, $q_{in} = 0$

$\therefore (E_{in})_A = (E_{in})_B = 0$

$V = \frac{\sigma R}{\epsilon_0}$

And, $V_A = V_B$

$\therefore \sigma_A R_A = \sigma_B R_B \longrightarrow \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$



$V = C$

$\therefore \frac{QR}{4\pi R^2} = C$

$\frac{Q}{4\pi R} = C$

$\therefore Q \propto R$

$\propto Q_A > Q_B$

$E = \frac{V}{R}$

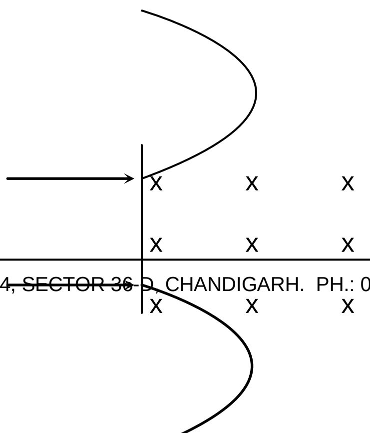
$\therefore E \propto \frac{1}{R}$

$E_A < E_B$ on surface

$\therefore$ All are correct

34. (b,d) They will follow ↑↑ (parallel) path

$t = \frac{2\pi m}{Bq}$



But $m_e \neq m_e$ +

$\therefore\ t_{e^-} \neq t_{e^+}$

(b) and (d) are correct

35. (d)

36. (c) $v = \omega\sqrt{A^2 - x^2}$

$P = m\omega\sqrt{A^2 - x^2}$

$P^2 = m^2\omega^2 (A^2 - x^2)$

$P^2 + m^2\omega^2x^2 = m^2\omega^2A^2$

Circle $\Rightarrow m\omega^2 = 1$

$P^2 + x^2 = A^2$

$\Rightarrow \quad P^2 + x^2 = 2mE$

$\Rightarrow \quad$ Radius $R^2 = 2mE$

$R = \sqrt{2mE}$

$\Rightarrow \sqrt{\frac{E_1}{E_2}} = \frac{2a}{a}$

$\Rightarrow \frac{E_1}{E_2} = 4$

Also $E = \frac{1}{2}m\omega^2A^2$

$\Rightarrow E = \frac{A^2}{2m}$

37. (b)

38. (c) Given that

$N = [M^oL^{-3}T^o]$ $\quad m = [M\,L^o\,T^o]$

$e = [M^oL^oTI]$

$E_0 = [I^2\,T^2]$

$E_0 = [M^{-1}\,L^{-3}\,T^4\,I^2]$

$\therefore \sqrt{\frac{Ne^2}{m\in_0}} = \sqrt{\frac{[L^{-3}][T^2I^2]}{[M][M^{-1}L^{-3}T^4I^2]}}$

$= \sqrt{T^{-2}} = [T^{-1}]$

39. (c) $\lambda = \frac{c}{\nu}$, $w = 2\pi\nu$

$\sqrt{\frac{Ne^2}{m\in_0}} = 2\pi\nu$

$\sqrt{\frac{Ne^2}{m\in_0}} = 2\pi\frac{c}{\lambda}$

$\lambda = 2\pi c\sqrt{\frac{m\in_0}{Ne^2}}$

$\lambda = 2\pi \times 3 \times 10^8 \sqrt{\frac{10^{-30} \times 10^{-11}}{4 \times 10^{27} \times (1.6 \times 10^{-19})^2}}$

$\lambda = 2\pi \times 3 \times 10^8 \sqrt{\frac{10^{-41}}{4 \times 10^{27-38} \times (1.6)^2}}$

$$\lambda = 2\pi \times 3 \times 10^8 \sqrt{\frac{10^{-41}}{4\times10^{-11}(1.6)^2}}$$

$$\lambda = 2\pi \times 3 \times 10^8 \sqrt{\frac{10^{-41+11}}{4\times(1.6)^2}}$$

$$\lambda = 2\pi \times 3 \times 10^8 \sqrt{\frac{10^{-30}}{4\times(1.6)^2}}$$

$$\lambda = \frac{2\times3.14\times3\times10^8\times10^{-15}}{2\times1.6}$$

$\lambda = 5.88 \times 10^{-7} = 5888 \text{ nm} \approx 600 \text{ nm}$

40. (3) $F_{14} = \frac{kq^2}{a^2}$

$F_{12} = \frac{kq^2}{a^2} \quad \Rightarrow F_{13} = \frac{kq^2}{2a^2}$

F resultant on 1 charge = $\frac{kq^2}{2a^2} + \frac{\sqrt{2}kq^2}{a^2} = \frac{Ya}{4}$

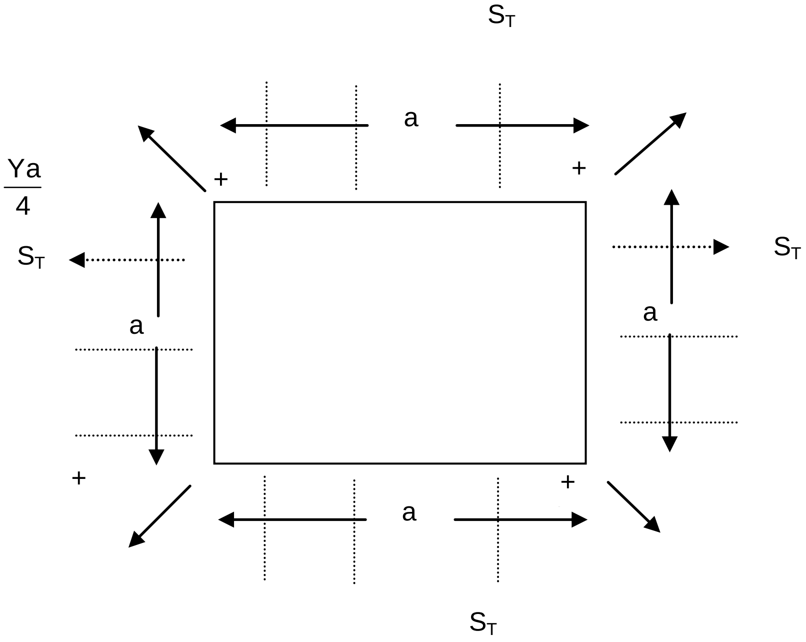
$\therefore$ N= 3

$= \left(\frac{La^2}{Y}\left(2+4\sqrt{2}\right)\right)^{1/3} = a$

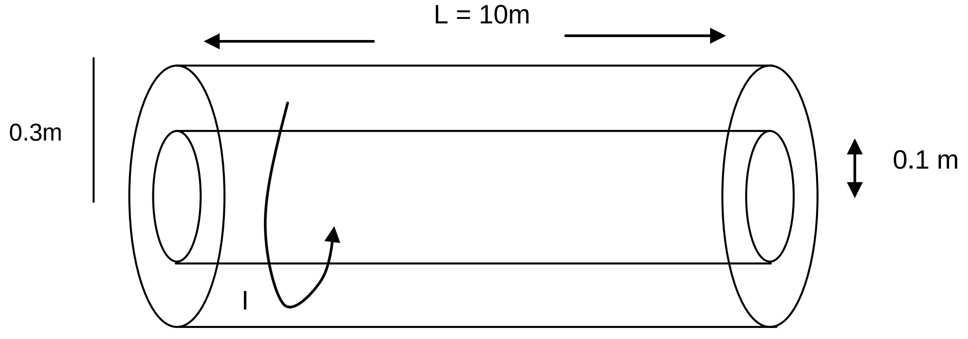
$= \frac{kq^2}{a^2}\left(\frac{1}{2}+\sqrt{2}\right) = \frac{Ya}{4}$

$= \frac{4kq^2}{a^2}\left(\frac{1}{2}+\sqrt{2}\right) = Ya$

$= \frac{kq^2}{y}\left(2+4\sqrt{2}\right) = a^3$



41. (6)



Magnetic field due to the tube (r = 0.3 m)

$B = \mu o n I \qquad \left(n \to \frac{\text{no. of turns}}{\text{length}}\right)$

$\therefore B = \frac{\mu o I}{10}$

$B = \dfrac{\mu_0 I_0 \ \text{Cos}\,300t}{10}$

Flux linked with tube of radius = 0.1m

$\phi = B \times \pi \times (0.1)^2$

$\phi = \pi\, b \times 0.01$

$\phi = \dfrac{\pi \mu_0 I_0 \,\text{Cos}\,300t}{10} \times 0.01$

$\therefore$ E.m. f induced will be

$$e = -\frac{d\phi}{dt} = -1\left(\frac{d}{dt}\frac{(\pi \mu_0 I_0 \,\text{Cos}\,300t \times 0.01)}{10}\right)$$

$$e = -\left(\frac{\pi \ \mu_0 I_0 \times 300 \times 0.01 \ \sin 300t}{10}\right)$$

$$e = \frac{3\pi \mu_0 I_0 \sin 300t}{10}$$

$\therefore$ Induced current is

$$I = \frac{e}{R} = \frac{3\pi \ \mu_0 I_0 \ \sin 300t}{0.005 \times 10}$$

$$I = \frac{3 \times 1000 \times \pi \times \mu_0 I_0 \ \sin 300t}{5 \times 10}$$

Magnetic Moment associated

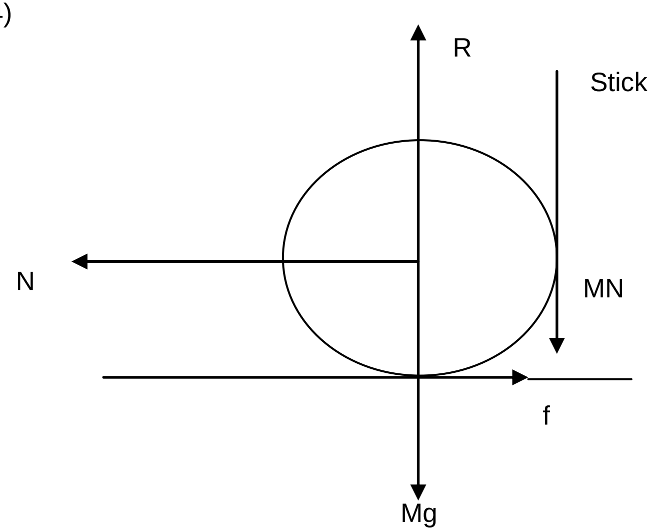
$M \quad = IA = 60 \times \pi \ \mu_0 \ I_0 \sin 300\ t \times \pi \times (0.1)^2$

$\qquad = 60 \times \pi^2 \ \mu_0 \ I_0 \ \sin 300\ t \times \dfrac{1}{100}$

$\pi^2 \approx 10 \ = \dfrac{600}{100} \ \mu_0 \ I_0 \ \sin 300\ t$

$\qquad = 6\ \mu_0\ I_0 \ \sin 300t \qquad\qquad \therefore N = 6$

42. (4)



According to F = ma

N- f = M a

or N- f = 2 (.3) = .6 …………..(1)

and by T = Ia

$(f - MN)(.5) = 2(.5)^2 \dfrac{a}{R}$

or $(f - MN)\dfrac{1}{2} = 2\,\dfrac{1}{4}.\dfrac{3}{5}$

or $f - MN = \dfrac{3}{5}$ ……………(2)

adding (1) & (2)

$N - MN = \dfrac{6}{5}$

Since N = 2 and $M = \dfrac{P}{10}$

$2 - \dfrac{2p}{10} = \dfrac{6}{5}$

$\Rightarrow \dfrac{P}{10} = \dfrac{4}{10}$ $\Rightarrow P = 4$

43. (9) $I_1 = I_2 = \dfrac{2}{5}Mr^2$

$I_2 = \dfrac{2}{5} \times \dfrac{1}{2} \times \dfrac{5}{4} = \dfrac{1}{4} = I_1$

$I_2 + I_1 = \dfrac{1}{2}$ kg cm^2

$I_2 = I_4 = \dfrac{2}{5}MR^2 + Md^2$

$= \dfrac{2}{5}\times\dfrac{1}{2}\times\dfrac{5}{4} + \dfrac{1}{2} \times 4 \times 2$

$= \left(\dfrac{1}{9} + 4\right)$ kg cm^2

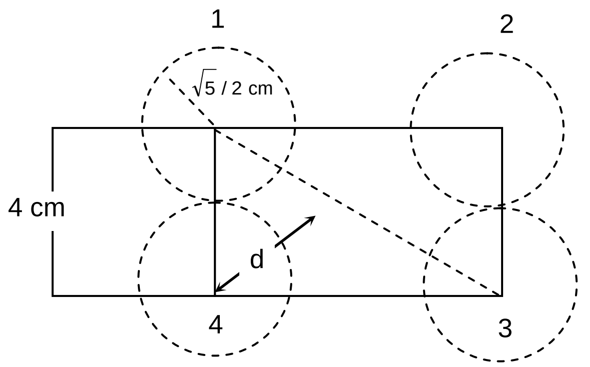
$I_2 + I_4 = \dfrac{1}{2} + 8$ kg cm^2

$\therefore I_1 + I_2 + I_3 + I_4 = \left(\dfrac{1}{2} + \dfrac{1}{2} + 8\right)$ kg cm^2

= 9 kg cm^2

= 9×10^{-4} kg m^2

$\therefore$ N is 9



44. (1) $\dfrac{dN}{dt} = \lambda N$

i.e $10^{10} = \dfrac{1}{T_M}N$

$\Rightarrow 10^{10} = \dfrac{1}{10^9}N$

Hence number of atoms n $=10^{19}$

Mass of radioactive sample

= (mass of 1 atom)$\times$ no. of 1 atoms

=1 mg

45. (5) $mg(\sin\theta + \mu\cos\theta) = 3\,mg(\sin\theta - \mu\cos\theta)$

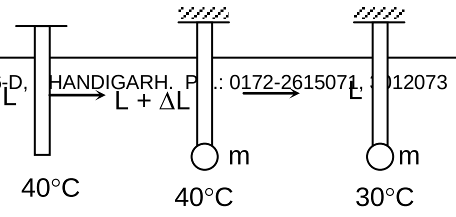
$\sin\theta + \mu\cos\theta = 3\sin\theta - 3\mu\cos\theta$

$4\mu\cos\theta = 2\sin\theta$

$\mu = \dfrac{1}{2}\tan\theta = \dfrac{1}{2}$

$\Rightarrow N = 10\mu = 5$

46. (3) $y = \dfrac{F/A}{DL/L} \Rightarrow \dfrac{\Delta L}{L}y = \dfrac{F}{A}$



$\Rightarrow \Delta L = \frac{FL}{Ay} = \frac{mgL}{Ay}$

Also $\Delta L = L \propto \Delta T$

$\Rightarrow \frac{mgL}{Ay} = L \propto \Delta T$

$m = \frac{Ay \propto \Delta T}{g} = \frac{\pi(10^{-3})^2.10^{11}.10^{-15}.10}{10}$

$= \pi$

$m \cong 3$ kg

47. (c) xi + yi + zk

from (c) option,

$$[\vec{a}\ \vec{b}\ \vec{r}] = 0 \text{ and } \frac{\vec{r}\cdot\vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{3}}$$

where $\vec{r}$ is the required vector.

48. (d) $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$

$\sin\theta - \cos\theta = \sqrt{2}\cos\theta$

$\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} = \sqrt{2}$

$\tan\theta - 1 = \sqrt{2}$

$\tan\theta = \sqrt{2} + 1$

$\tan\theta = \tan\frac{3\pi}{8} \Rightarrow \theta = n\pi + \frac{3\pi}{8}.\ n \in Z$

$\theta = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\sin\theta$

$\frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \sqrt{2}$

$1 + \cot\theta = \sqrt{2}$

$\cot\theta = \sqrt{2} - 1$

$\Rightarrow \cot\theta = \cot\frac{3\pi}{8} \Rightarrow \theta = m\pi + \frac{3\pi}{8}.\ m \in Z \Rightarrow P = Q$

49. (b)

$\int_0^b (x-1)^2 dx - \int_b^1 (x-1)^2 dx = \frac{1}{4}$

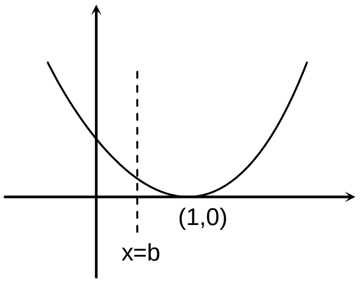
$\frac{1}{3}\left|(x-1)^3\right|_0^b - \frac{1}{3}\left|(x-1)^3\right|_b^1 = \frac{1}{4}$

$[(b-1)^3 - (-1)] - [0 - (b-1)^3] = \frac{3}{4}$

$(b-1)^3 + 1 + (b-1)^3 = \frac{3}{4}$

$2(b-1)^3 = -\frac{1}{4}$

$(b-1)^3 = -\frac{1}{8} \Rightarrow b - 1 = \frac{-1}{2} \Rightarrow b = \frac{1}{2}$



50. (c) $\alpha^2 - 6\alpha - 2 = 0$

$\alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$

$\beta^{10} - 6\beta^9 - 2\beta^8 = 0$

$(\alpha^{10} - \beta^{10}) - 6(\alpha^9 - \beta^9) - 2(\alpha^8 - \beta^8) = 0$

$a_{10} - 6a_9 - 2a_8 = 0$

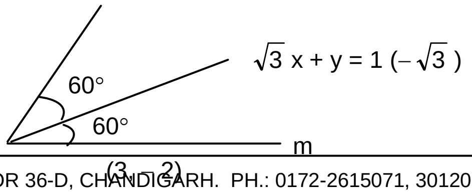
$\frac{a_{10} - 2a_8}{2a_9} = 3$

51. (b)

$y + 2 = \sqrt{3}(x-3)$

$y + 2 = \sqrt{3}x - 3\sqrt{3}$

$y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$



$\Rightarrow \left|\frac{m-(-\sqrt{3})}{1+\sqrt{3}(-m)}\right| = \tan 60^\circ$

$\frac{m+\sqrt{3}}{1-\sqrt{3}m} = \sqrt{3}, -\sqrt{3}$

$m + \sqrt{3} = \sqrt{3} - 3m, -\sqrt{3} + 3m$

$4m = 0, -2m = -2\sqrt{3}$

$m = \sqrt{3}$

52. (c) $(2x)^{\ell n2} = (3x)^{\ell n3}$

$2^{\ell n2}x^{\ell n2} = 3^{\ell n3}.y^{\ell n3}$

$2^{\ell n2+\ell nx} = 3^{\ell n3+\ell ny}$

$\ell n2 + \ell nx = 0, \ell n3 + \ell n\, y = 0$

$x = \frac{1}{2}, y = \frac{1}{3}$, which satisfies second equation

$3^{\ell n\frac{1}{2}} = 2^{\ell n\frac{1}{3}}$

$\ell n\frac{1}{2}\ \ell n3 = \ell n\frac{1}{3}\ \ell n2$

53. (a) $\int_{\sqrt{\ell n2}}^{\sqrt{\ell n3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ell n_6 - x^2)}\, dx$

Put $x^2 = t$

$I = \frac{1}{2}\int_{\ell n_2}^{\ell n_3} \frac{\sin t\, dt}{\sin t + \sin(\ell n_6 - t)}$ ....(1)

$I = \frac{1}{2}\int_{\ell n_2}^{\ell n_3} \frac{\sin(\ell n_6 - t)dt}{\sin t + \sin(\ell n_6 - t)}$ ....(2)

$2I = \frac{1}{2}\int_{\ell n_2}^{\ell n_3} 1.\, dt$

$2I = \frac{1}{2}\ \ell n\frac{3}{2}$

$I = \frac{1}{4}\ell n\ \frac{3}{2}$

54. (b,d) $\frac{x^2}{4} + \frac{y^2}{1} = 1$

$a = 2, b = 1$

$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{1}{4} = \frac{3}{4}$

$e = \frac{\sqrt{3}}{2}$; $F(\pm\sqrt{3}, 0)$

$\therefore e' = \frac{2}{\sqrt{3}}$ is eccentricity of

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

As it passes through $(\pm\sqrt{3}, 0)$

$\frac{3}{a^2} = 1 \Rightarrow a^2 = 3$

$b^2 = a^2\,(e^2 - 1)$

$= 3\left(\frac{4}{3} - 1\right) = 1$

$\therefore \frac{x^2}{3} - \frac{y^2}{1} = 1$

$x^2 - 3y^2 = 3$

55. (b,c) Let $f(x + y) = f(x) + f(y)\ \forall x, y \in R$

$f(x) = xf(1)$

56. (a,d)

(a) $(\hat{i} + \hat{j} + \hat{k})\,.\,(\hat{j} - \hat{k}) = 1 - 1 = 0$

(b) $(\hat{i} + \hat{j} + \hat{k})\,.\,(-\hat{i} + \hat{j}) = 0$

(c) $(\hat{i} + \hat{j} + \hat{k})\,.\,(\hat{i} - \hat{j}) = 0$

(a) $\begin{vmatrix} 0 & 1 & -1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = -1\,(1 - 2) - 1\,(2 - 1) = +1 - 1 = 0$

(b) $\begin{vmatrix} -1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = -1\,(1 - 4) - (1 - 2) = 3 + 1 = 4 \neq 0$

(c) $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 1\,(1 - 4) + 1(1 - 2) = -3 - 1 \neq 0$

(d) $\begin{vmatrix} 0 & -1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 1\,(1 - 2) + (2 - 1) = -1 + 1 = 0$

57. (c) $M^T = -M,\ N^T = -N$

$MN = NM$

$M^2N^2\,(M^TN)^{-1}(MN^{-1})^T$

$M^2N^2\,N^{-1}\,(-M)^{-1}\,(N^{-1})^T\,M^T$

$M^2N\,[-M^{-1}]\,(N^T)^{-1}\,(-M)$

$-M^2N\,M^{-1}\,N^{-1}\,M$

$-M\,M\,N\,M^{-1}\,N^{-1}\,M$

$-M\,N\,N^{-1}\,M$

$-M^2$

58. (d) $[a\ b\ c]\ \begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix}$

$a = \lambda,\ b = 6\lambda,\ c = -7\lambda$

$a + 8b + 7c = 0$

$9a + 2b + 3c = 0$

$a + b + c = 0$

$2(\lambda) + 6\lambda - 7\lambda = 1$

$\lambda = 1$

$7a + b + c = 7(1) + 6 - 7 \Rightarrow 6$

59. (a) $\omega = \frac{-1 + \sqrt{3}i}{2}$

$a = 2,\ \lambda = 2$

b = 12, c = –14

$\frac{3}{\omega^2}+\frac{1}{\omega^{12}}+\frac{3}{\omega^{-14}}$

$3\omega + 1 + 3\omega^2$

$3(\omega + \omega^2) + 1 = 1 - 3 = -2$

Use $\omega^{14} = \omega^{12} . \omega^2 = \omega^2$

60. (b) $a = \lambda,\ b = 6\lambda,\ c = -7\lambda$

$\lambda = 1,\ a = 1,\ b = 6,\ c = -7$

$x^2 + 6x - 7 = 0$

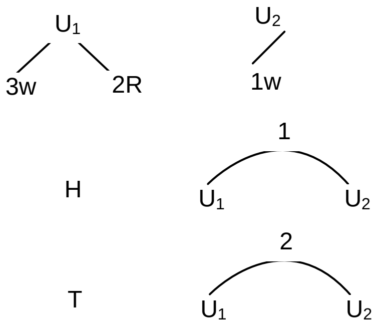
$(x + 7)(x - 1) = 0$

$x = 1, -7$

Using, $\left(1-\frac{1}{7}\right)^n = \left(\frac{6}{7}\right)^n$

$\sum_{n=1}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n = \frac{1}{1-\frac{6}{7}} = 7$

61. (b)



E_1 = Head appear

E_2 = Tail appear

A – Draw ball from U_2 being white

$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)$

$= \frac{1}{2} \times \left[\frac{3}{5}\times 1+\frac{2}{5}\times\frac{1}{2}\right] + \frac{1}{2}\times\left[\frac{{}^3C_2}{{}^5C_2}\times 1+\frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3}+\frac{{}^3C_1\times{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right]$

$= \frac{1}{2}\times\left[\frac{4}{5}\right]+\frac{1}{2}\left[\frac{3}{10}+\frac{1}{30}+\frac{2}{5}\right]$

$= \frac{2}{5}+\frac{1}{2}\left[\frac{9+1+12}{30}\right]$

$= \frac{2}{5}+\frac{1}{2}\times\frac{22}{30}$

$= \frac{2}{5}+\frac{11}{30}$

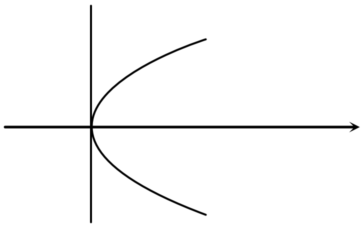
$= \frac{12+11}{30}=\frac{23}{30}$

62. (d) $P(E_1/A) = \frac{P(E_1)P(A/E_1)}{23/30}=\frac{2/5}{23/30}=\frac{12}{23}$

63. (7) n = 7 (Using C–D and A–B formulas)

64. (8) $\frac{a^{-5}+a^{-4}+a^{-3}+a^{-3}+a^{-3}+1+a^8+a^{10}}{8} \geq (1)^{\frac{1}{8}}$ (AM ≥ GM)

65. (2)



$4a = 8, a = 2$

$(a, 2a)\ (a, -2a)$

$(2, 4)\ (2, -4)$

$= \frac{1}{2}\begin{vmatrix} 2 & 4 & 1 \\ 2 & -4 & 1 \\ 1/2 & 2 & 1 \end{vmatrix}$

$= \frac{1}{2}|2(-6) - 4(3/2) + (4+2)|$

$= \frac{1}{2}|-12 - 6 + 6| = 6$

$\Delta_1 = 6$

$yy_1 = 2a(x + x_1)$

$yy_1 = 4(x + x_1)$

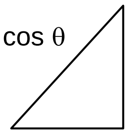
$4y = 4(x + 2) \qquad -4y = 4(x + 2)$

$y = x + 2 \qquad -y = x + 2$

$x - y + 2 = 0 \qquad x + y + 2 = 0$

Area of triangle formed by tangents is 3

66. (1) $f(\theta) = \sin\left[\tan^{-1}\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right]$



$= \sin[\sin^{-1}(\tan\theta)]$

$= \tan\theta$

$\frac{df(\theta)}{d(\tan\theta)} = 1$

67. (5) $|z - (3 + 2i)| \le 2$

$|z_1 - z_2| \ge |z_1| - |z_2|$

$|2z - 6 + 5i|$

$|2z - 6 - 4i + 4i + 5i|$

$|2(z - 3 - 2i) + 9i| \ge ||9i| - 2|z - 3 - 2i|| \qquad (|z_1 - z_2| \ge |z_1| - |z_2|)$

$\ge |9 - 2(2)|$

$\ge |5|$

68. (9) $a_1 = 3$

$s_p = a_1 + a_2 + a_3 + \ldots + a_p$

$= 3 + (3 + d) + (3 + 2d) + \ldots + 3 + (P - 1)d$

$= 3p + d\frac{p(p-1)}{2}$

$\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{15n + d\frac{5n(5n-1)}{2}}{3n + \frac{dn(n-1)}{2}}$

$\frac{30n + 5nd(5n-1)}{6n + nd(n-1)}$

$\frac{30n + 25n^2d - 5nd}{6n + n^2d - nd}$

put $d = 6$

$\frac{30n + 150n^2 - 30n}{6n + 6n^2 - 6n} = 25$

For $d = 6$ $\frac{S_m}{S_n}$ is independent of n

$a_2 = 3 + 6 = 9$

$a_2 = 9$

69. (6) $6\int_1^x f(t)dt = 3xf(x) - x^3$

$6f(x) = 3[xf'(x) + f(x)] - 3x^2$

$2y = x\frac{dy}{dx} + y - x^2$

$x\frac{dy}{dx} - x^2 = y$

$x\frac{dy}{dx} - y = x^2$

$\frac{dy}{dx} - \frac{y}{x} = x$

$\text{I.F} = e^{\int -\frac{1}{x}dx} = e^{-\ell nx} = \frac{1}{x}$

$y.\frac{1}{x} = \int x.\frac{1}{x}\,dx + c$

$\frac{y}{x} = x + c$

$y = x^2 + cx$

$2 = 1 + c,$ $c = 1$

$y = x^2 + x$

$y = 2^2 + 2 = 6$